Competing physical processes in turbulent fluid dynamics

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When there are two globally conserved quantities in a turbulent fluid, the dynamics of one can be influenced significantly by the dynamics of the other. This is a familar feature in two-dimensional turbulence where conservation both of kinetic energy and of a quantity known as the *enstrophy* forces a net kinetic energy transfer towards large scales and a net enstrophy transfer towards the small scales. In three-dimensional turbulence, both kinetic energy and *helicity* are quadratic invariants. Helicity is the measure of parity-breaking (helical or *twisting*) motions in the fluid. The kinetic energy transfer processes have been thought to dominate the dynamics at all scales with the helicity being carried along passively. We showed that helicity possesses a timescale for transfer which can affect the energy transfer rate. Consequently, our understanding of two key features of turbulent fluid dynamics needs to be revised.

The only constraint between helicity and kinetic energy in a given wavenumber k is given by the Schwartz inequality for the *relative* helicity :

$$\mathcal{H}(k) = \frac{H(k)}{2kE(k)} \leq 1$$

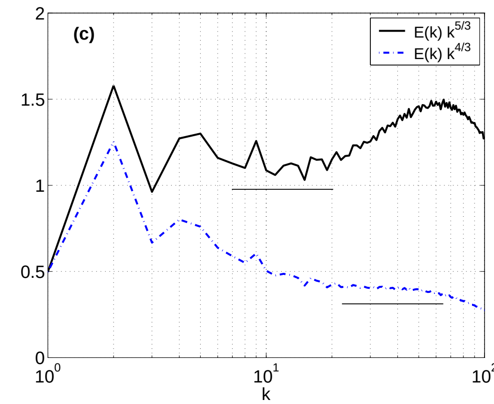
where $H(k)$ is the helicity density and $E(k)$ the kinetic energy density in wavenumber k. This relation is the main argument used to justify neglecting helicity. The relation seems to imply that for sufficiently high wavenumber the relative helicity $\mathcal{H}(k)$ must go to zero and hence the helicity effects must become vanishingly small. This is somewhat misleading as we describe below.

Denote the kinetic energy transfer timescale, that is, the time taken for energy to be fluxed through wavenumber k, by τ_E and the corresponding timescale for helicity by τ_H. Using simple arguments based on locality of the transfer processes in wavenumber [1] one can define these timescales and deduce that their ratio is:

$$\frac{\tau_E}{\tau_H} \leq \left(\frac{H(k)}{2kE(k)}\right)^{1/2} = \mathcal{H}(k)^{1/2}$$

That is, the ratio of the relevant timescales governing the dynamics falls of *slower* than the relative helicity $\mathcal{H}(k)$. So, while the energy transfer is always the faster, and hence the more dominant, timescale, we cannot rule out the fact the helicity timescale may not be that much slower and could in fact affect the overall dynamics.

The first consequence of this study of timescales of the two quadratic invariants is that a $k^{-4/3}$ scaling of the energy spectrum $E(k)$ becomes possible if the helicity transfer timescale is not too slow. This is a fundamental revision of the



The energy spectrum as a function of wavenumber as computed from statistically steady turbulent flow in a periodic cube with 1024 grid points to a side. The solid black curve is the spectrum compensated by $k^{5/3}$. The region $7 < k < 20$ indicates the nominal range over which the Kolmogorov $k^{-5/3}$ scaling holds. The dashed blue line is the same spectrum compensated by $k^{-4/3}$. The flat regime now occurs for $20 < k < 70$, showing that a transition between the energy timescale dominated dynamics and the helicity timescale dominated dynamics has a signature in the energy spectrum.

decades old Kolmogorov benchmark that the energy spectrum must scale as $k^{-5/3}$ for highly turbulent flows. The figure shows the energy spectrum calculated from data generated by direct numerical simulation of the Navier-Stokes equation with random helical forcing in a periodic box with 1024^3 grid points to a side. In order to identify the two possible scaling regimes the spectrum is compensated by $k^{5/3}$ (solid black curve) and by $k^{4/3}$ (dashed blue curve). In the former, compensation by $k^{5/3}$ reveals a flat regime $6 < k < 20$ and followed by the well-known 'bottleneck' feature often observed in turbulence spectrum measurements. There is an apparent 'pile-up' of energy in the wavenumbers past a nominal $k^{-5/3}$ scaling regime. Remarkably, the bottleneck disappears when the spectrum is compensated by $k^{-4/3}$ suggesting that there is a reasonable explanation for the bottleneck – that the energy is slowed down by helicity and hence appears to 'pile-up' in the large wavenumbers before dissipating.

A further consqeuence of a timescale comparison is that we deduced a new dissipation scale for both energy and helicity which is larger than the dissipation scale prescribed by Kolmogorov. Traditional computational requirements for numerical simulation of turbulence, have always aimed to resolve the Kolmogorov energy dissipation scale. The prediction that a larger, helicity dependent dissipation scale exists requires further verification but if true, implies that we only need to resolve something significantly larger than the Kolmogorov scale. This implies a potentially significant saving in resources when very large flows need to be numerically computed.

This report is a summary of work published in [2].

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